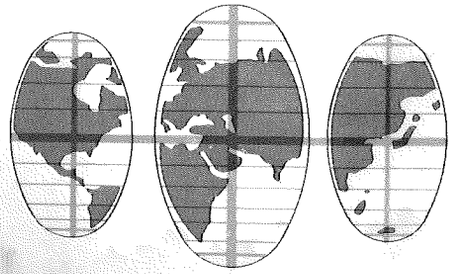

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$$du^2 = \frac{\partial x}{\partial x}\,du^2 + \frac{\partial x}{\partial u}\,du\,du \left(\frac{\partial x}{\partial x}\right.$$

/∂x and ∂f/∂y are each funct
o each of them as follows:

$$\left(\frac{\partial f}{\partial x}\right) = \frac{\partial(\partial f/\partial x)}{\partial x}\frac{dx}{du} + \frac{\partial(\partial f/\partial y)}{\partial y}\frac{dy}{du}$$

$$\left(\frac{\partial f}{\partial y}\right) = \frac{\partial(\partial f/\partial y)}{\partial x}\frac{dx}{dv} + \frac{\partial(\partial f/\partial$$

-13) ma

2.64	2.55	2.48	2.41	2.32	2.24	2.1								
1.07	1.92	1.88	1.85	1.70	1.75	1 68								
2.59	2.51	2.43	2.36	2.26	2.19	2.06	1.98	1.89	1.79	1.73	1.64	1.59	1.51	1
1.95	1.90	1.86	1.83	1.77	1.72	1.65	1.60	1.55	1.49	1.45	1.39	1.36	1.31	
2.56	2.47	2.40	2.33	2.25	2.15	2.03	1.94	1.85	1.75	1.68	1.59	1.54	1.46	
.94	1.89	1.85	1.82	1.76	1.71	1.64	1.59	1.54	1.47	1.44	1.37	1.34	1.20	
.53	2.44	2.37	2.30	2.20	2.12	2.00	1.91	1.83	1.72	1.66	1.56	1.51	1.43	
2	1.87	1.83	1.80	1.74	1.69	1.62	1.57	1.52	1.45	1.42	1.35	1.32	1.26	
	2.41	2.34	2.28	2.17	2.09	1.97	1.88	1.79	1.69	1.62	1.53	1.48	1.39	
	.85	1.81	1.78	1.72	1.67	1.00	1.54	1.49	1.42	1.38	1.32	1.28	1	
		87	2.29	2.22	2.12	2.04	1.92	1.84	1.74	1.64	1.57	1.47	1.42	
				1.80	1.76	1.70	1.65	1.58	1.53	1.47	1.41	1.36	1.30	1
				86	2.20	2.09	2.01	1.89	1.81	1.71	1.61	1.54	1.44	

Potential flow about

used. At $z \to \infty$,
ie r direction is V_r.

$-kV_0 \cdot (l \cos\theta \sin\phi$

at $z = r \cos\phi \to$

$\sin\phi$

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HAROLD S. MICKLEY

Professor of Chemical Engineering
Massachusetts Institute of Technology

THOMAS K. SHERWOOD

Professor of Chemical Engineering
Massachusetts Institute of Technology

CHARLES E. REED

Engineering Manager, Chemical Department
General Electric Company

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PREFACE

Applied mathematics has assumed increasing importance to the chemical engineer. The authors of current professional literature freely use transform, vector, and finite-difference methods to attack a problem. The practicing engineer is finding modern mathematical techniques a valuable tool in the analysis of a variety of situations. Present trends in the chemical and process industries involve increased emphasis on automatic-control systems, high-speed machine computation, operational analysis, and the like. These developments directly depend upon the application of advanced mathematical procedures.

The purpose of this book is to consolidate the advanced methods of mathematics into a form that can be applied readily by both the student and the professional engineer. Emphasis is focused on the engineering applications of mathematics. Considerable attention is given to the problem of expressing a physical situation in mathematical language. Problems drawn from the engineering literature are used to illustrate the mathematical procedures. The material covered falls into three broad categories: (1) the treatment and interpretation of engineering data, (2) the analysis of situations involving only one independent variable, and (3) the analysis of situations involving two or more independent variables.

The mathematical background of the reader is presumed to be limited. The text material begins with a discussion of the process of differentiation, and the development of more advanced procedures follows in a step-by-step manner.

The book may be used as either an undergraduate or a graduate text. The material of Chaps. 1, 3, 4, and 5 can be handled readily by the undergraduate and will provide the background needed for the assimilation of the more advanced material in a subsequent study program.

The second edition represents an extensive revision of the original work. Chapter 8 (The Laplace Transform), Chap. 9 (Analysis of Stagewise Processes by the Calculus of Finite Differences), and Chap. 10 (The Numerical Solution of Partial Differential Equations) are completely new. The remaining chapters have been rewritten, and new material has been

added. Such topics as the statistics of small samples, analysis of variance, factorial design of experiments, expansion in a series of orthogonal functions, vector notation, and others have been included.

The decision to add new material and to omit material formerly included has been difficult. The action taken has been based upon an appraisal of trends both in engineering education and in the engineering profession.

The treatment of the material used has been influenced by a number of sources. Primary impetus was supplied by the first edition of "Applied Mathematics in Chemical Engineering." Marshall and Pigford's "The Application of Differential Equations to Chemical Engineering Problems" focused attention on the utility of the Laplace transform and the calculus of finite differences in the analysis of the transient behavior of unit operations equipment. The advanced mathematics courses given at the Massachusetts Institute of Technology, and in particular those that have led to F. B. Hildebrand's books, "Advanced Calculus for Engineers" and "Methods of Applied Mathematics," have been of great help. The texts by R. V. Churchill, "Fourier Series and Boundary Value Problems," "Modern Operational Mathematics in Engineering," and "Introduction to Complex Variables and Applications," have been consulted frequently. Other major sources of material have been listed in the bibliography at the end of each chapter.

The authors are indebted to Professor Sir Ronald A. Fisher, Cambridge, and to Oliver & Boyd, Ltd., Edinburgh and London, for permission to reprint Table 2-1 from their book "Statistical Methods for Research Workers,"; to Professor P. C. Mahalanobis, F.R.S., Calcutta, and to the Indian Statistical Institute, Calcutta, for permission to reprint Table 2-4 from an article appearing in *Sankhyā;* to Professor G. W. Snedecor, Ames, Iowa, and to Collegiate Press, Inc., of Iowa State College, Ames, for permission to reprint Table 2-5 from their book "Statistical Methods Applied to Experiments in Agriculture and Biology"; to Dr. T. v. Kármán and Dr. M. A. Biot and to McGraw-Hill Book Company, Inc., New York, for permission to reprint Table 5-2 from their book "Mathematical Methods in Engineering"; to Professor R. V. Churchill, Ann Arbor, Mich., and to McGraw-Hill Book Company, Inc., New York, for permission to reprint Table 8-1 from their book "Modern Operational Mathematics in Engineering." The constructive suggestions and encouragement of the author's professional colleagues are gratefully acknowledged. In particular, the invaluable aid in the preparation of the second edition by the authors of the first edition, Professor T. K. Sherwood and Dr. C. E. Reed, is sincerely appreciated.

HAROLD S. MICKLEY

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