H. Edelsbrunner
Algorithms in Combinatorial Geometry

This book offers a modern approach to computational geometry, an area that studies the computational complexity of geometric problems. Computational geometry is strongly connected to the considerably older field of combinatorial geometry. For example, the combinatorial structure of a geometric problem usually decides which algorithmic method solves the problem most efficiently. Furthermore, the analysis of an algorithm often requires a great deal of combinatorial knowledge.

The book consists of three parts: a combinatorial part, a computational part, and one that presents applications of the results of the first two parts. The choice of the topics covered in this book was guided by the attempt to describe the most fundamental algorithms in computational geometry that have an interesting combinatorial structure. The book is also a collection of a large number of results giving the book an encyclopedic character. Each chapter includes a set of exercises of various degrees of difficulty. One of the purposes of these collections of exercises and open problems is to give results that extend the material presented in the corresponding chapters; another purpose is to point out related open problems. Each chapter also contains a collection of bibliographic notes.

ISBN 3-540-13722-X
ISBN 0-387-13722-X

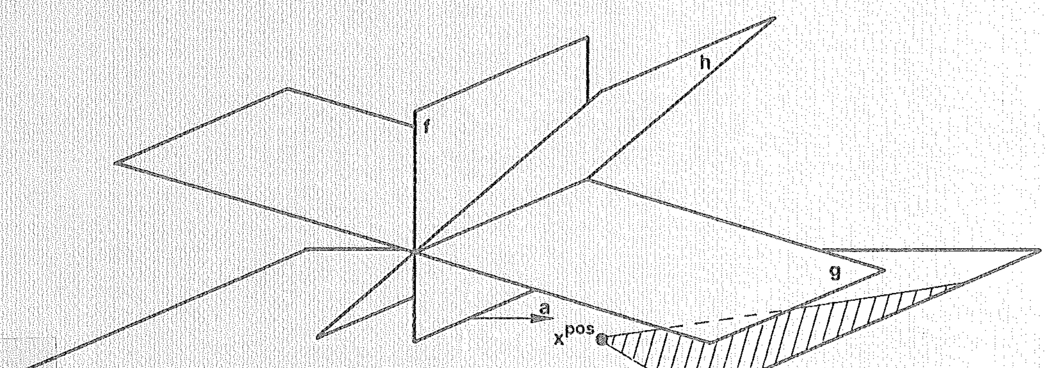
EATCS 10

Edelsbrunner Algorithms in Combinatorial Geometry

EATCS
Monographs on Theoretical Computer Science
W. Brauer G. Rozenberg A. Salomaa (Eds.)

HERBERT EDELSBRUNNER

Algorithms in Combinatorial Geometry



Springer-Verlag

EATCS
Monographs on Theoretical Computer Science
Volume 10

Editors: W. Brauer G. Rozenberg A. Salomaa

EATCS Monographs on Theoretical Computer Science

Vol. 1: K. Mehlhorn: Data Structures and Algorithms 1: Sorting and Searching. XIV, 336 pages, 87 figs. 1984.

Vol. 2: K. Mehlhorn: Data Structures and Algorithms 2: Graph Algorithms and NP-Completeness. XII, 260 pages, 54 figs. 1984.

Vol. 3: K. Mehlhorn: Data Structures and Algorithms 3: Multidimensional Searching and Computational Geometry. XII, 284 pages, 134 figs. 1984.

Vol. 4: W. Reisig: Petri Nets. An Introduction. X, 161 pages, 111 figs. 1985.

Vol. 5: W. Kuich, A. Salomaa: Semirings, Automata, Languages. IX, 374 pages, 23 figs. 1986.

Vol. 6: H. Ehrig, B. Mahr: Fundamentals of Algebraic Specification 1. Equations and Initial Semantics. XI, 321 pages. 1985.

Vol. 7: F. Gécseg: Products of Automata. VIII, 107 pages, 18 figs. 1986.

Vol. 8: Kröger: Temporal Logic of Programs. VIII, 148 pages. 1987.

Vol. 9: K. Weihrauch: Computability. X, 517 pages. 1987.

Vol. 10: H. Edelsbrunner: Algorithms in Combinatorial Geometry. XV, 423 pages, 93 figs. 1987.

Herbert Edelsbrunner

Algorithms in Combinatorial Geometry

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo

Author

Prof. Dr. Herbert Edelsbrunner
University of Illinois at Urbana-Champaign
Dept. of Computer Science
1304 West Springfield Avenue, Urbana, Illinois 61801, USA

Editors

Prof. Dr. Wilfried Brauer
Institut für Informatik, Technische Universität München
Arcisstrasse 21, 8000 München 2, Germany

Prof. Dr. Grzegorz Rozenberg
Institute of Applied Mathematics and Computer Science
University of Leiden, Niels-Bahr-Weg 1, P.O. Box 9512
2300 RA Leiden, The Netherlands

Prof. Dr. Arto Salomaa
Department of Mathematics, University of Turku
20500 Turku 50, Finland

ISBN 3-540-13722-X Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-13722-X Springer-Verlag New York Berlin Heidelberg

Library of Congress Cataloging-in-Publication Data. Edelsbrunner, Herbert. Algorithms in combinatorial geometry. (EATCS monographs on theoretical computer science ; v. 10). Bibliography: p. 1. Combinatorial geometry. 2. Geometry–Data processing. I. Title. II. Series. QA167.E34 1987 516'.13 87-16655
ISBN 0-387-13722-X (U.S.)

Offsetprinting: Druckhaus Beltz, Hemsbach
Binding: J. Schäffer GmbH & Co. KG, Grünstadt
2145/3140-543210

to my wife Eva

PREFACE

Computational geometry as an area of research in its own right emerged in the early seventies of this century. Right from the beginning, it was obvious that strong connections of various kinds exist to questions studied in the considerably older field of combinatorial geometry. For example, the combinatorial structure of a geometric problem usually decides which algorithmic method solves the problem most efficiently. Furthermore, the analysis of an algorithm often requires a great deal of combinatorial knowledge. As it turns out, however, the connection between the two research areas commonly referred to as computational geometry and combinatorial geometry is not as lop-sided as it appears. Indeed, the interest in computational issues in geometry gives a new and constructive direction to the combinatorial study of geometry.

It is the intention of this book to demonstrate that computational and combinatorial investigations in geometry are doomed to profit from each other. To reach this goal, I designed this book to consist of three parts, a combinatorial part, a computational part, and one that presents applications of the results of the first two parts. The choice of the topics covered in this book was guided by my attempt to describe the most fundamental algorithms in computational geometry that have an interesting combinatorial structure. In this early stage geometric transforms played an important role as they reveal connections between seemingly unrelated problems and thus help to structure the field. These transforms led me to believe that arrangements of hyperplanes are at the very heart of computational geometry – and this is my belief now more than ever.

As mentioned above, this book consists of three parts: **I. Combinatorial Geometry**, **II. Fundamental Geometric Algorithms**, and **III. Geometric and Algorithmic Applications.** Each part consists of four to six chapters. The non-trivial connection pattern between the various chapters of the three parts can be somewhat untangled if we group the chapters according to four major computational problems. The construction of an arrangement of hyperplanes is tackled in Chapter 7 after Chapters 1, 2, and 5 provide preparatory investigations. Chapter 12 is a collection of applications of an algorithm that constructs an arrangement. The construction of the convex hull of a set of points which is discussed in Chapter 8 builds on combinatorial results presented in Chapter 6. Levels and other structures in an arrangement can be computed by methods described in Chapter 9 which bears a close relationship to the combinatorial studies undertaken in Chapter 3. Finally, space cutting algorithms are presented in Chapter 14 which is based on the combinatorial investigations of Chapter 4 and the computational results of Chapter 10. The above listing of relations between the various chapters is by no means exhaustive. For example, the connections between Chapter 13 and the other chapters of this book come in too many shapes to be described here. Finally, Chapter 15 reviews the techniques used in the other chapters of this book to provide some kind of paradigmatic approach to solving computational geometry problems.

Prerequisites together with notational conventions followed in this book are collected at the end in Appendices A and B. Each chapter includes a set of exercises of various degrees of difficulty. I have tried to estimate the difficulty of each problem and expressed my opinion in terms of numbers 1 through 5 with the meaning defined as follows:

1 if the problem is trivial or very easy,
2 if the problem is easy but it may be tedious to solve it,
3 if the problem is of moderate difficulty,
4 if the problem is very difficult, and
5 if the problem is still unsolved; this does not necessarily mean that it is very difficult.

Of course, the assignment of these numbers is purely subjective except for the number 5 which is used to mark research problems. One of the purposes of these collections of exercises and open problems is to give results that extend the material presented in the corresponding chapters; another purpose is to point out related open problems. Each chapter also contains a collection of bibliographic notes which occasionally give pointers to places in the literature where solutions to some of the more difficult problems in the exercise section can be found.

I would like to acknowledge the help of many colleagues and friends without whom this book would never have been written. I thank Raimund Seidel for providing the original notes for Section 8.4 and for thoroughly reading earlier versions of parts of the book. I also thank Carlos Bhola, Jeffrey Salowe, Emo Welzl, and an anonymous referee who suffered through earlier versions of all chapters and provided many valuable suggestions, comments, and corrections. Thanks also to Bernard Chazelle, Friedrich Huber, Ernst Mücke, Harald Rosenberger,

and Steven Skiena for carefully reading earlier versions of various chapters. Many of the algorithms presented in this book have been implemented during projects at the Technical University of Graz and the University of Illinois at Urbana-Champaign. For these implementations I thank Barbara Geymayer, Michael Hirschböck, Friedrich Huber, Hartwig Huemer, Tom Madej, Ernst Mücke, Harald Rosenberger, Gerd Stöckl, and Roman Waupotitsch who sacrificed many of their valuable hours to do the job. For discussions on topics found in this book, I thank Franz Aurenhammer, Bernard Chazelle, David Dobkin, Jacob Goodman, Branko Grünbaum, Leonidas Guibas, David Haussler, David Kirkpatrick, Hermann Maurer, Kurt Mehlhorn, Ernst Mücke, Joseph O'Rourke, Janos Pach, Richard Pollack, Franco Preparata, Harald Rosenberger, Jean-Pierre Roudneff, Franz Josef Schnitzer, Raimund Seidel, Micha Sharir, William Steiger, Gerd Stöckl, Jan van Leeuwen, Roman Waupotitsch, Emo Welzl, Douglas West, Derick Wood, and Frances Yao. I am also grateful to Hans Wössner and Gillian Hayes from Springer-Verlag, Heidelberg, for the pleasure it was to work with them. Last, but not least, I thank Heidrun Kaiser at Graz and Janet Shonkwiler and June Wingler at Urbana for valuable assistance in typing this book.

My most particular thanks go to my wife Eva whose patience and encouragement as well as help in preparing this book made it possible for me to finish the task.

Urbana, May 1987 — Herbert Edelsbrunner

TABLE OF CONTENTS

PART I

COMBINATORIAL GEOMETRY

The art of counting and estimating is at the heart of combinatorics – and it is a necessary prerequisite for analyzing algorithms and for deciding which algorithms are the most efficient ones. Part I of this book presents several combinatorial geometry problems and solutions using a variety of techniques.