This series reports new developments in computer science research and teaching – quickly, informally and at a high level. The type of material considered for publication includes preliminary drafts of original papers and monographs, technical reports of high quality and broad interest, advanced level lectures, reports of meetings, provided they are of exceptional interest and focused on a single topic. The timeliness of a manuscript is more important than its form which may be unfinished or tentative. If possible, a subject index should be included. Publication of Lecture Notes is intended as a service to the international computer science community, in that a commercial publisher, Springer-Verlag, can offer a wide distribution of documents which would otherwise have a restricted readership. Once published and copyrighted, they can be documented in the scientific literature.

Manuscripts

Manuscripts should be no less than 100 and preferably no more than 500 pages in length.
They are reproduced by a photographic process and therefore must be typed with extreme care. Symbols not on the typewriter should be inserted by hand in indelible black ink. Corrections to the typescript should be made by pasting in the new text or painting out errors with white correction fluid. Authors receive 75 free copies and are free to use the material in other publications. The typescript is reduced slightly in size during reproduction; best results will not be obtained unless the text on any one page is kept within the overall limit of 18 x 26.5 cm (7 x 10½ inches). On request, the publisher will supply special paper with the typing area outlined.
Manuscripts should be sent to Prof. G. Goos, GMD Forschungsstelle an der Universität Karlsruhe, Haid- und Neu-Str. 7, 7500 Karlsruhe 1, Germany, Prof. J. Hartmanis, Cornell University, Dept. of Computer Science, Ithaca, NY/USA 14850, or directly to Springer-Verlag Heidelberg.

Springer-Verlag, Heidelberger Platz 3, D-1000 Berlin 33
Springer-Verlag, Tiergartenstraße 17, D-6900 Heidelberg 1
Springer-Verlag, 175 Fifth Avenue, New York, NY 10010/USA
Springer-Verlag, 37-3, Hongo 3-chome, Bunkyo-ku, Tokyo 113, Japan

ISBN 3-540-52055-4
ISBN 0-387-52055-4

400 Klein Concrete and Abstract Voronoi Diagrams

C.N.R.
ISTITUTO per la MATEMATICA APPLICATA
BIBLIOTECA
INVENTARIO
N° 11274
QA278

Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

400

Rolf Klein

Concrete and Abstract Voronoi Diagrams

Springer-Verlag

Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

400

Rolf Klein

Concrete and Abstract Voronoi Diagrams

Springer-Verlag
Berlin Heidelberg New York London Paris Tokyo Hong Kong

Editorial Board

D. Barstow W. Brauer P. Brinch Hansen D. Gries D. Luckham
C. Moler A. Pnueli G. Seegmüller J. Stoer N. Wirth

Author

Rolf Klein
Universität-Gesamthochschule-Essen, Fachbereich Mathematik
Praktische Informatik mit Schwerpunkt Softwareorientierte Informatik
Schützenbahn 70, D-4300 Essen 1, FRG

CR Subject Classification (1987): E.1, F.2.2, H.3.3, I.3.5

ISBN 3-540-52055-4 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-52055-4 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication of this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its version of June 24, 1985, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1989
Printed in Germany

Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.
2145/3140-543210 – Printed on acid-free paper

Contents

Preface 1

Introduction 3

1 Voronoi diagrams in nice metrics 9

1.1 The Euclidean Voronoi diagram . . . 9
1.2 Nice metrics . . . 16

2 Abstract Voronoi diagrams 31

2.1 Definitions . . . 32
2.2 Elementary properties . . . 34
2.3 The neighborhoods of points in V(S) . . . 36
2.4 Augmented curve systems . . . 43
2.5 The shape of the Voronoi regions . . . 46
2.6 The graph structure of abstract Voronoi diagrams . . . 51
2.7 Characterizing Voronoi diagrams . . . 52

3 Computing abstract Voronoi diagrams 63

3.1 Representing the Voronoi diagram . . . 64
3.2 The divide-and-conquer approach . . . 67
3.3 Bisecting chains . . . 73
3.4 The non-degenerate case . . . 77
3.4.1 Determining endpoints of chain segments . . . 77
3.4.2 Finding the continuing segment . . . 92
3.4.3 The complete algorithm . . . 97
3.5 The degenerate case . . . 99
3.5.1 Local separation of curves . . . 101
3.5.2 Cross-points . . . 109
3.5.3 The modified algorithm . . . 112

4 Acyclic partitions 131
4.1 Supported d-circles 131
4.2 Simply-connected d-circles 140
4.3 The Moscow metric 147

Concluding Remarks 159

Bibliography 163

Preface

The Voronoi diagram is one of the most useful data structures in computational geometry, with applications in many areas of science. In this book, a unifying approach is proposed by introducing abstract Voronoi diagrams. These are not based on the notion of distance but rather on the concept of bisecting curves.

This book is a revised version of my Habilitationsschrift *On a generalization of planar Voronoi diagrams* that was accepted by the Faculty of Mathematics, University of Freiburg, FRG, in February 1989. I have included some recent developments, and have brought up to date the selection of open problems in the concluding remarks.

I want to thank the Institute of Computer Science, the Faculty of Mathematics, and the University of Freiburg, for their support. Special thanks to Christine Krause and Lihong Ma, who did an excellent job in LaTeXing the manuscript!

I am grateful to Walter Nef, Hartmut Noltemeier, Joachim Nitsche, Thomas Ottmann, Peter Widmayer, and John Wieacker, who gave their opinion on this work, and to the anonymous referees. Their interest, good will, and patience be thanked!

A great debt of gratitude I owe to Anne Brüggemann-Klein, Kurt Mehlhorn, and Derick Wood. Derick came up with the topic of proximity problems in cities and, thereby, gave rise to this work. He also joined in, during the last weeks of my stay with the Data Structuring Group in Waterloo in 1987. Kurt encouraged me to publish these Lecture Notes. His appreciation was as rewarding as the fact that he, together with Colm Ó'Dúnlaing and Stefan Meiser, continued the research begun in my thesis. Without Anne's support, this book would not be the same. The discussions with her helped me to clarify my thoughts, she cheered me up when I needed it, and—she drew the figures in this text. Thanks to all of you!

Essen, October 1989 Rolf Klein